Fuzzy Inf. Eng. (2015) 7: 115–128

HOSTED BY

ELSEVIER

Available online at www.sciencedirect.com

ScienceDirect

Fuzzy Information and Engineering

http://www.elsevier.com/locate/fiae

ORIGINAL ARTICLE

Centrality Measures in Directed Fuzzy Social Networks

CrossMark

Ren-Jie Hu · Qing Li · Guang-Yu Zhang · Wen-Cong Ma

Received: 20 December 2013/ Revised: 10 February 2015/
Accepted: 25 February 2015/

Abstract The centrality of actors has been a key issue in undirected fuzzy social network (UFSN) analysis. For undirected fuzzy social networks (UFSNs) where edges are just a present or absent undirected fuzzy relation with no more information attached. There has been a growing need to design centrality measures for directed fuzzy social networks (DFSNs), because DFSNs where edges are attached with directed fuzzy relation would contain rich information. In this paper, we propose some new centrality measure called fuzzy in-degree centrality, fuzzy out-degree centrality, fuzzy in-closeness centrality and fuzzy out-closeness centrality which are applicable to the DFSNs. It unveils more structural information about directed fuzzy relation and connectivity of DFSNs. Furthermore, by investigating the validness and robustness of this new centrality measure by illustrating this method to some cases and we obtain reliable results, which provide strong evidences of the new measures' utility.

Keywords Directed fuzzy relation · Directed fuzzy social network · Fuzzy in-degree centrality · Fuzzy in-closeness centrality

---

Ren-Jie Hu (✉) · Qing Li
Guangdong Research Institute for International Strategies, Guangdong University of Foreign Studies, Guangzhou 510420, China
email: renjiehu2005@163.com
Guang-Yu Zhang · Wen-Cong Ma
School of Management, Guangdong University of Technology, Guangzhou 510520, China

Peer review under responsibility of Fuzzy Information and Engineering Branch of the Operations Research Society of China.

http://dx.doi.org/10.1016/j.fiae.2015.03.008

1. Introduction

A social network is a set of nodes representing people, groups, organizations, enterprises, etc., which are connected by links showing relations or flows between them. Social network analysis (SNA) is used to study the implications of the restrictions of different actors in their communications and then in their opportunities of relation. The fewer constraints an actor face, the more opportunities he/she will have, and thus he will be in a more favorable position to bargain in exchanges and to intermediate in the bargains of others that need him/her, to increase his/her influence.

In SNA, a problem of determining the importance of actors in a network has been studied for a long time [1]. It is in this context that the concept of the centrality of a vertex emerged in a network. The SNA considers closely related concepts of centrality and power as individuals in, which informs us of aspects about the facts who is in the network, who acts as a leader, who is an intermediary, who appears almost isolated, who stands central, or, who looks peripheral.

Social networks researchers have developed several centrality measures. Degree, closeness and betweenness centralities are without doubt the three most popular ones. Degree centrality focuses on the level of communication activity, identifying the centrality of a node with its degree [2, 3]. Closeness centrality considers the sum of the geodesic distances between a given actor and the remaining as a centrality measure in the sense that the lower this sum is, the greater the centrality [4, 5]. Closeness centrality is, then, a measure of independence in communications, in relations or in bargaining, and thus, it measures the possibility to communicate with many others depending on a minimum number of intermediaries. Betweenness centrality emphasizes the value of communication control: the possibility to intermediate in the relation of others [6, 7]. Here, all possible geodesic paths are considered between pairs of nodes. The centrality of each actor is the number of such paths in which it lies.

Centrality analysis is used extensively in social and behavioral sciences, as well as science in politics, management, economics, biology, and so on. Stephenson and Zelen [8] abandoned the geodesic path as structural element in the definition of centrality, introduced a measure based on the concept of information as it is used in the theory of statistical estimation. The defined measure uses a weighted combination of all paths between pairs of nodes, the weight of each path depending on the information contained in it. Bonacich [9, 10] suggested another concept of centrality. He proposed measuring the centrality of different nodes by using the eigenvector associated with the largest characteristic eigenvalue of the adjacent matrix. Brunelli et al. [11] introduced a flexible consensus measure that is taken into account in the influence strength of decision makers according to their eigenvector centrality. Sohn and Kim [12] developed a robust methodology to computing zone centrality measures in an urban area. Kermarrec et al. [13] introduced a novel form of centrality: the second order centrality which can be computed in a distributed manner. Shen and Chen [14] researched relationship between network centrality, absorptive capacity and innovation performance of small medium enterprise (SME). Ping and Zong [15] studied on micro-blog information dissemination based on SNA centrality analysis. Qi et al. [16] proposed a new centrality measure called the Laplacian centrality measure for weighted networks. Laplacian centrality is an intermediate measuring between global and local characterization of the importance of a vertex. Unfortunately, Laplacian centrality also has a limitation that it cannot be applied in directed networks. Pozo et al. [17] defined a family of centrality measures as directed social networks from a game theoretical point of view. Landherr et al. [18] analyzed five common centrality measures on the basis of three simple requirements for the behavior of centrality measures.

The notion of a fuzzy social network (FSN) and the methods to fuzzy social network analysis (FSNA) have attracted considerable interest and curiosity from the social and behavioral science community in recent decades. Nair and Sarasamma [19] applied a fuzzy theory to perform social network analysis, defined UFSN as a fuzzy graph with the entities as the nodes or actors and the relations among them as the edges or links. Liao and Hu [20] defined the concept of UFSN and explored some of its basic properties. The definition and relevant analysis provide theoretical foundation for further study of the FSN. Fan et al. [21, 22] discussed structural equivalence and regular equivalence in UFSNs. Liao et al. [23] studied the position and role with the concepts of structural equivalence in the undirected fuzzy technology innovation network. Tseng [24] proposed fuzzy network balanced scorecard (FNBSC) as a performance evaluation method when the aspects and criteria are dependent and interaction is uncertain. Liao et al. [25] extended the notion of centrality to the fuzzy framework, proposing fuzzy degree centrality, fuzzy closeness centrality and fuzzy betweenness centrality in UFSN. Hu et al. [26] extended the notion of centrality and centralization to the fuzzy frameworks, suggesting fuzzy inversely attenuation closeness centrality, and discussing fuzzy group closeness centralization based on inversely attenuation factor in UFSNs.

However, the fuzzy relation between actors in FSN are assumed undirected until now. In order to study the problem of directed fuzzy relation between actors, Hu and Zhang [27] proposed directed fuzzy social network (DFSN), and studied fuzzy structural holes analysis in it. In this paper, we extend the notion of fuzzy centrality to DFSN, proposing some new centrality measure called fuzzy in-degree centrality, fuzzy out-degree centrality, fuzzy in-closeness centrality and fuzzy out-closeness centrality which are applicable to the DFSNs.

The remainder of this paper is organized as follows: Section 2 offers some preliminary concepts; In Section 3, we present fuzzy in-degree centrality, fuzzy out-degree centrality of DFSNs. In Section 4, fuzzy in-closeness centrality is described together with fuzzy out-closeness centrality of DFSNs. Analytical and numerical results are shown, based on various centrality measures applying on R-J Research Team in Section 5. And finally, a conclusion appears in Section 6.

2. Preliminaries

Many social network measures have been defined for binary situations in which a pair of actors is either connected or not. However, the relation between actors is generally characterized by vague notions, such as ‘equally’, ‘moderately’, ‘strongly’, ‘very strongly’, ‘extremely’ or ‘significant degree’. The fuzzy set theory, introduced by Zadeh [28, 29], is suitable for dealing with the uncertainty and imprecision associated with information concerning various parameters. Liao and Hu [20] applied fuzzy theory to social network, defined UFSN as follows.

Definition 2.1 [20] *A undirected fuzzy social network is defined as a fuzzy relational structure $\widetilde{G}_{un} = (V, \widetilde{E}_{un})$, where $V = \{v_1, v_2, \cdots, v_n\}$ is a non-empty set of actors or nodes, and* $\widetilde{E}_{un} = \begin{pmatrix} \widetilde{e_{11}} & \cdots & \widetilde{e_{1n}} \\ \vdots & \ddots & \vdots \\ \widetilde{e_{n1}} & \cdots & \widetilde{e_{nn}} \end{pmatrix}$ *is a undirected fuzzy relation on V.*

Many fuzzy relations are directional. A fuzzy relation is directional if the ties are oriented from one actor to another. So, Hu and Zhang [27] defined DFSN as follows.

Definition 2.2 [27] *A directed fuzzy social network is defined as a fuzzy relational structure $\widetilde{G}_{dn} = (V, \widetilde{E}_{dn})$, where $V = \{v_1, v_2, \cdots, v_n\}$ is a non-empty set of actors or nodes, and* $\widetilde{E}_{dn} = \begin{pmatrix} \widetilde{e_{11}} & \cdots & \widetilde{e_{1n}} \\ \vdots & \ddots & \vdots \\ \widetilde{e_{n1}} & \cdots & \widetilde{e_{nn}} \end{pmatrix}$ *is a undirected fuzzy relation on V.*

FSN includes UFSN and DFSN. The major difference between DFSN and UFSN lies in the fact that directed fuzzy relation is taken into consideration. According to Definition 2.1, $\widetilde{e_{ij}}$ is equal to $\widetilde{e_{ji}}$ in undirected fuzzy social network. However, $\widetilde{e_{ij}}$ is not always equal to $\widetilde{e_{ji}}$ in directed fuzzy social network. In Definition 2.2, $\widetilde{E}_{dn}$ is called directed fuzzy adjacency matrix of $\widetilde{G}_{dn}$. $\mu_{\widetilde{E}_{dn}} = \begin{pmatrix} \mu(\widetilde{e}_{11}) & \cdots & \mu(\widetilde{e_{1n}}) \\ \vdots & \ddots & \vdots \\ \mu(\widetilde{e_{n1}}) & \cdots & \mu(\widetilde{e_{nn}}) \end{pmatrix}$ is a membership function which is asymmetrical.

The related concepts of UFSNs can also be used in DFSNs, but the direction of the fuzzy relation between actors must be taken into consideration.

We now discuss how several of the concepts for social networks are applied to DFSNs. We will focus on the most important concepts of it including the directed fuzzy intensity, directed fuzzy connected intensity and directed fuzzy connected intensity matrix.

Definition 2.3 *Assume that $v_0e_1v_1e_2v_2\cdots e_kv_k$ is a path from v_0 to v_k in $\widetilde{G}_{dn}$, then $\widetilde{s}_d(\widetilde{\omega}) = \bigwedge_{i=1}^{k} \mu(e_i)$ is called directed fuzzy intensity of path $\widetilde{\omega}$.*

In Definition 2.3, $\mu(e_i)$ is a membership function.

Definition 2.4 *If $\widetilde{\omega}_k$ are n paths from u to v in $\widetilde{G}_{dn}$, then $\widetilde{s}_d(u, v) = \bigvee_{k=1}^{n} \widetilde{s}_d(\widetilde{\omega}_k)$ is called a directed fuzzy connected intensity from u to v.*

Here, $k = 1, 2, \cdots, n$. If there is no path from u to v, then $\widetilde{s}_d(u, v) = 0$. If $u = v$, then $\widetilde{s}_d(u, v) = 1$. In UFSN, $\widetilde{s}(u, v)$ always equals $\widetilde{s}(v, u)$. However, $\widetilde{s}_d(u, v)$ is not always equal to $\widetilde{s}_d(v, u)$ in DFSN.

Definition 2.5 *Assume that $\widetilde{G}_{dn} = (V, \widetilde{E}_{dn})$ is a DFSN*

$$\widetilde{C}_d = \begin{pmatrix} \widetilde{s}(v_1, v_1) \cdots & \widetilde{s}(v_1, v_n) \\ \vdots \quad \ddots & \vdots \\ \widetilde{s}(v_n, v_1) \cdots & \widetilde{s}(v_n, v_n) \end{pmatrix},$$

then $\widetilde{C}_d$ is called a directed fuzzy connected intensity matrix of $\widetilde{G}_{dn}$.

The directed fuzzy connected intensity matrix is a very important concept of DFSN. According to directed fuzzy connected intensity matrix, we can know the directed fuzzy connected intensity between any of two actors in DFSN, that is to say, we can know the relationship (direct or indirect) between any of two actors in DFSN.

3. The Fuzzy Degree Centrality Analysis of Directed Fuzzy Social Network

In this section, we present measurement of fuzzy in-degree centrality, fuzzy out-degree centrality and fuzzy degree centrality, respectively, in DFSN.

Definition 3.1 *Assume that $\widetilde{G}_{dn} = (V, \widetilde{E}_{dn})$ is a DFSN, $\widetilde{d}_I(v_i)$ is the sum of the fuzzy relations that are adjacent to v_i, then $\widetilde{d}_I(v_i)$ is called fuzzy in-degree centrality of v_i.*

According to Definition 3.1, the formula of fuzzy in-degree centrality of v_i is

$$\widetilde{d}_I(v_i) = \sum_{j=1, j\neq i}^{n} \widetilde{e}_{ji}. \tag{1}$$

Definition 3.2 *Assume that $\widetilde{G}_{dn} = (V, \widetilde{E}_{dn})$ is a DFSN, $\widetilde{d}_O(v_i)$ is the sum of the fuzzy relations that are adjacent from v_i, then $\widetilde{d}_O(v_i)$ is called fuzzy out-degree centrality of v_i.*

The formula of fuzzy out-degree centrality of v_i is

$$\widetilde{d}_O(v_i) = \sum_{j=1, j\neq i}^{n} \widetilde{e}_{ji}. \tag{2}$$

Definition 3.3 *Assume that $\widetilde{G}_{dn} = (V, \widetilde{E}_{dn})$ is a DFSN, $\widetilde{d}(v_i)$ is the sum of $\widetilde{d}_I(v_i)$ and $\widetilde{d}_O(v_i)$, then $\widetilde{d}(v_i)$ is called fuzzy out-degree centrality of v_i.*

The formula of fuzzy degree centrality of v_i is

$$\widetilde{d}(v_i) = \widetilde{d}_I(v_i) + \widetilde{d}_O(v_i). \tag{3}$$

In DFSN applications, these degrees can be of great interest. The fuzzy out-degree centralities are measures of expansiveness and the fuzzy in-degree centralities are measures of receptivity, or popularity.

Definition 3.4 *Assume that $\widetilde{G}_{dn} = (V, \widetilde{E}_{dn})$ is a DFSN, then $\overline{d}_I = \frac{\sum_{i=1}^{n} \widetilde{d}_I(v_i)}{n}$ is called mean fuzzy in-degree centrality of $\widetilde{G}_{dn}$.*

Definition 3.5 *Assume that* $\widetilde{G}_{dn} = (V, \widetilde{E}_{dn})$ *is a DFSN, then* $\overline{d}_O = \frac{\sum_{i=1}^{n} \widetilde{d}_O(v_i)}{n}$ *is called mean fuzzy out-degree centrality of* $\widetilde{G}_{dn}$.

Remark 3.1 *Assume that* $\widetilde{G}_{dn} = (V, \widetilde{E}_{dn})$ *is a DFSN,* $\overline{d}_I$ *is mean fuzzy in-degree centrality of* $\widetilde{G}_{dn}$*, and* $\overline{d}_O$ *is mean fuzzy out-degree centrality of* $\widetilde{G}_{dn}$*, then* $\overline{d}_I = \overline{d}_O$.

Proof Assume that $\widetilde{G}_{dn} = (V, \widetilde{E}_{dn})$ is a DFSN. Since the fuzzy in-degree centralities count the fuzzy relations incident from the actors, and the fuzzy out-degree centralities count the fuzzy relations incident to the actors, $\sum_{i=1}^{n} = \widetilde{d}_I(v_i) = \sum_{i=1}^{n} \widetilde{d}_O(v_i)$, and thus we can see $\overline{d}_I = \overline{d}_O$.

One might also be interested in variability of the fuzzy in-degree centralities and fuzzy out-degree centralities. Unlike the mean fuzzy in-degree centralities and the mean fuzzy out-degree centralities, the variance of the fuzzy in-degree centralities is not necessarily the same as the variance of the fuzzy out-degree centralities.

The variance of the fuzzy in-degree centrality, which we denote by $S^2_{\widetilde{d}_I}$, is calculated as

$$S^2_{\widetilde{d}_I} = \frac{\sum_{i=1}^{n} [\widetilde{d}_I(v_i) - \overline{d}_I]^2}{n}. \tag{4}$$

Similarly, the variance of the fuzzy out-degree centrality, which we denote by $S^2_{\widetilde{d}_O}$, is calculated as

$$S^2_{\widetilde{d}_O} = \frac{\sum_{i=1}^{n} [\widetilde{d}_O(v_i) - \overline{d}_O]^2}{n}. \tag{5}$$

$S^2_{\widetilde{d}_I}$ and $S^2_{\widetilde{d}_O}$ measures quantify how unequal the actors in a DFSN are with respect to initiating or receiving fuzzy relations. These measures are simple statistics for summarizing how "centralized" a DFSN is.

In the DFSNs, fuzzy in-degree centralities are measures of the acceptance or popularity of actors, and fuzzy out-degree centralities are measures influence of actors. In terms of the fuzzy in-degree centrality and fuzzy out-degree centrality there are four possible kinds of actors: The actor is an isolate, the actor only has fuzzy relation originating from it, the actor only has fuzzy relation terminating at it, or the actor has fuzzy relation both to and from it. According to this classification, a actor is a(n):

- Isolate if $\widetilde{d}_I(v_i) = \widetilde{d}_O(v_i) = 0$,
- Transmitter if $\widetilde{d}_I(v_i) = 0, \widetilde{d}_O(v_i) > 0$,
- Receiver if $\widetilde{d}_I(v_i) > 0, \widetilde{d}_O(v_i) = 0$,
- Carrier of ordinary if $\widetilde{d}_I(v_i) > 0, \widetilde{d}_O(v_i) > 0$.

4. The Fuzzy Closeness Centrality Analysis of Directed Fuzzy Social Network

Liao et al. [25] assumed that any of two actors are reachable and proposed fuzzy closeness centrality in the UFSNs. Fuzzy closeness centrality considers the sum of the fuzzy relation between a given actor and the remaining as a centrality measure in the sense that the more this sum is, the greater the centrality. However, requirement of any of two actors who are reachable is too rigorous in the DFSN. Hence, how to extend the notion of fuzzy closeness centrality to DFSN is significant.

Definition 4.1 *Assume that $\widetilde{G}_{dn} = (V, \widetilde{E}_{dn})$ is a DFSN, $\widetilde{C}_{CI}(v_i)$ is the sum of the fuzzy connected intensity from all the actors (not include v_i) to v_i, then $\widetilde{C}_{CI}(v_i)$ called fuzzy in-closeness centrality of v_i.*

According to Definition 4.1, the formula of fuzzy in-closeness centrality of v_i is

$$\widetilde{C}_{CI}(v_i) = \sum_{j=1, j\neq i}^{n} \widetilde{s}(v_j, v_i). \tag{6}$$

Definition 4.2 *Assume $\widetilde{G}_{dn} = (V, \widetilde{E}_{dn})$ to be a DFSN, $\widetilde{C}_{CO}(v_i)$ is the sum of the fuzzy connected intensity from actor v_i to all the other actors, then $\widetilde{C}_{CO}(v_i)$ is called fuzzy out-closeness centrality of v_i.*

According to Definition 4.2, the formula of fuzzy out-closeness centrality of v_i is

$$\widetilde{C}_{CO}(v_i) = \sum_{j=1, j\neq i}^{n} \widetilde{s}(v_i, v_j). \tag{7}$$

Definition 4.3 *Assume that $\widetilde{G}_{dn} = (V, \widetilde{E}_{dn})$ is a DFSN, $\widetilde{C}(v_i)$ to be the sum of $\widetilde{C}_{CI}(v_i)$ and $\widetilde{C}_{CO}(v_i)$, then $\widetilde{C}(v_i)$ i is called fuzzy closeness centrality of v_i.*

The formula of fuzzy closeness centrality of v_i is

$$\widetilde{C}_{DC}(v_i) = \widetilde{C}_{CI}(v_i) + \widetilde{C}_{CO}(v_i) = \sum_{j=1, j\neq i}^{n} \widetilde{s}(v_i, v_j) + \sum_{j=1, j\neq i}^{n} \widetilde{s}(v_j, v_i). \tag{8}$$

Definition 4.4 *Suppose that $\widetilde{G}_{dn} = (V, \widetilde{E}_{dn})$ is a DFSN, then $\overline{d}_{CI} = \frac{\sum_{i=1}^{n} \widetilde{C}_{CI}(v_i)}{n}$ is called mean fuzzy in-closeness centrality of $\widetilde{G}_{dn}$.*

Definition 4.5 *Assume that $\widetilde{G}_{dn} = (V, \widetilde{E}_{dn})$ is a DFSN, then $\overline{d}_{CO} = \frac{\sum_{i=1}^{n} \widetilde{C}_{CO}(v_i)}{n}$ is called mean fuzzy out-closeness centrality of $\widetilde{G}_{dn}$.*

One might also be interested in the variability of fuzzy in-closeness centrality or fuzzy out-closeness centrality. Unlike the mean fuzzy in-closeness centrality and the mean fuzzy out-closeness centrality, the variance of the fuzzy in-closeness centrality is not necessarily the same as the variance of the fuzzy out-closeness centrality.

The variance of the fuzzy in-closeness centrality, which we denote by $S^2_{\widetilde{C}_{CI}}$, is calculated as

$$S^2_{\widetilde{C}_{CI}} = \frac{\sum\limits_{i=1}^{n}[\widetilde{C}_{CI}(v_i) - \overline{d}_{CI}]^2}{n}. \tag{9}$$

Similarly, the variance of the fuzzy out-closeness centrality, which we denote by $S^2_{\widetilde{C}_{CO}}$, is calculated as

$$S^2_{\widetilde{C}_{CO}} = \frac{\sum\limits_{i=1}^{n}[\widetilde{C}_{CO}(v_i) - \overline{d}_{O}]^2}{n}. \tag{10}$$

$S^2_{\widetilde{C}_{CI}}$ and $S^2_{\widetilde{C}_{CO}}$ measures quantify how unequal the actors in a directed fuzzy social network are with respect to initiating or receiving direct and indirect fuzzy relations. These measures are simple statistics for summarizing how "centralized" a DFSN is.

Fuzzy in-closeness centrality reflects not only the actor's acceptance or popularity by directly fuzzy relation, but also reflects the actor's acceptance or popularity by indirectly fuzzy relation. Fuzzy out-closeness centrality reflects not only an actor's directly influence but also an actor's indirect influence. The major difference between fuzzy closeness centrality and fuzzy degree centrality lies in the fact that indirect fuzzy relationship is taken into consideration.

5. Case Study

$\widetilde{G}_{d8} = (V, \widetilde{E}_{d8})$ is a directed fuzzy communication network of the G-Y Research Team, $V = \{v_1, v_2, \cdots, v_8\}$ is a set of 8 researchers, $\widetilde{E}_{d8}$ is directed fuzzy relation among the 8 researchers. We got the fuzzy communication relationship between 8 researchers of G-Y Research Team by investigation as follows

$$\mu_{\widetilde{E}_{d8}} = \begin{pmatrix} 1 & 0.3 & 0 & 0.8 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0.6 & 0.9 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0.4 & 0.2 & 0 \\ 0 & 0.1 & 0 & 0 & 1 & 0.7 & 0 & 0 \\ 0 & 0 & 0 & 0.5 & 0.7 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0.2 & 0 & 0 & 1 \end{pmatrix}.$$

We used Netdraw in UCINET to draw the directed fuzzy communication network of the G-Y Research Team (ties have values>0), see Fig. 1.

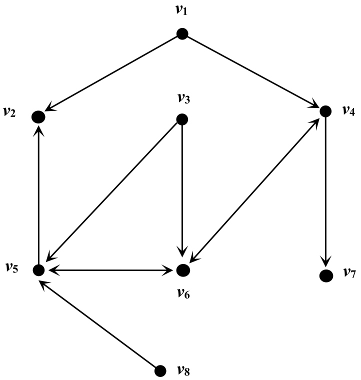

Fig. 1 Directed fuzzy communication network of the G-Y Research Team

According to Definition 2.5, we got a directed fuzzy connected intensity matrix of the G-Y Research Team as follows

$$\mu_{\widetilde{C}_{d8}} = \begin{pmatrix} 1 & 0.3 & 0 & 0.8 & 0.4 & 0.4 & 0.2 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0.1 & 1 & 0.5 & 0.7 & 0.9 & 0.2 & 0 \\ 0 & 0.1 & 0 & 1 & 0.4 & 0.4 & 0.2 & 0 \\ 0 & 0.1 & 0 & 0.5 & 1 & 0.7 & 0.2 & 0 \\ 0 & 0.1 & 0 & 0.5 & 0.7 & 1 & 0.2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0.1 & 0 & 0.2 & 0.2 & 0.2 & 0.2 & 1 \end{pmatrix}.$$

According to Eqs.(1)-(3), we got fuzzy in-degree centrality, fuzzy out-degree centrality and fuzzy degree centrality of the G-Y Research Team (see Table 1).

The scores based on the three are listed in Table 1. The top two actors are also indicated in bold. We also present detailed information of these 8 researchers based on three centrality measures in Fig. 2, where the X-axis represents these 8 researchers, and Y-axis corresponds to their scores. As we will see, based on fuzzy in-degree centrality, v_6 and v_5 get the two highest scores. It means that many other researchers nominate v_6 and v_5 as a friend. While v_1, v_3 and v_8 get the three lowest scores based on fuzzy in-degree centrality. It means these researchers are hardly welcomed by others. Based on fuzzy out-degree centrality, v_3 and v_6 get the two highest scores, the value of v_2 and v_7 are equal to 0. It means that v_3 and v_6 nominates many others as friends, but v_2 and v_7 have no influence on others. Based on fuzzy degree centrality, v_6 get the highest scores. It means that v_6 is liked by other actors with great influence on other actors.

Table 1: The fuzzy degree centrality of G-Y Research Team.

Researchers	Fuzzy in-degree centrality	Fuzzy out-degree centrality	Fuzzy degree centrality
v_1	0%	1.1%	1.1%
v_2	0.4%	0%	0.4%
v_3	04%	1.5%	1.5%
v_4	1.3%	0.6%	1.9%
v_5	1.4%	0.8%	2.2%
v_6	2%	1.2%	3.2%
v_7	0.2%	0%	0.2%
v_8	0%	0.2%	0.2%

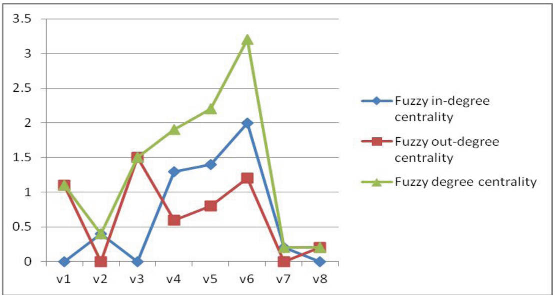

Fig. 2 Scatter diagram marked with straight lines and data based on three centrality methods to G-Y Research Team

According to Eqs.(6)-(8), the scores based on the three are listed in Table 2.

Listed in Table 2 are the scores based on all centrality methods. The top two actors are also indicated in bold. Furthermore, we also present detailed information of these 8 actors based on three centrality measures in Fig.3, where the X-axis represents these 8 researchers, and Y-axis corresponds to their scores. As we will see, based on fuzzy in-closeness centrality, v_6 and v_4 get the two highest scores. These two researchers have good interpersonal relationship and higher acceptance lies in the fact that indirect friendship is taken into consideration. Based on fuzzy out-degree centrality, v_3 and v_1 get the two highest scores. It means that these two researchers nominate many others as friends.

Furthermore, we also present detailed information of these 8 researchers based on various centrality methods in Fig. 4, where the X-axis represents these 8 researchers, and Y-axis corresponds to their scores. As we will see, based on fuzzy degree centrality and fuzzy closeness centrality, v_6 and v_5 get the two highest scores. It means that v_6 and v_5 is liked by other actors and has great influence on other actors.

Table 2: The fuzzy degree centrality of G-Y Research Team.

Researchers	Fuzzy in-closeness centrality	Fuzzy out-closeness centrality	Fuzzy closeness centrality
v_1	0%	2.1%	2.1%
v_2	0.8%	0%	0.8%
v_3	0%	2.4%	2.4%
v_4	2.5%	1.1%	3.6%
v_5	2.4%	1.5%	3.9%
v_6	2.6%	1.5%	4.1%
v_7	0.2%	0%	0.2%
v_8	0%	0.9%	0.9%

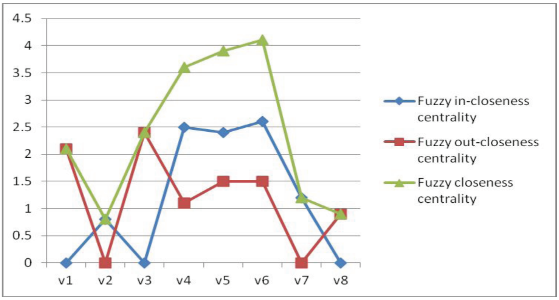

Fig. 3 Scatter diagram marked with straight lines and data based on three centrality methods to G-Y Research Team

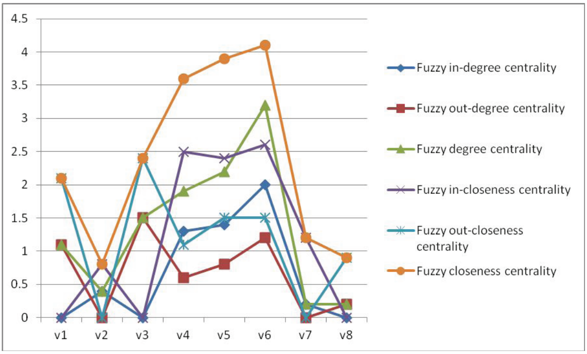

Fig. 4 Scatter diagram marked with straight lines and data based on various centrality methods to G-Y Research Team

6. Conclusion

In this paper, we extend a fuzzy centrality theory to DFSN, with some of its basic properties explored.

Firstly, we define the concept of directed fuzzy intensity, directed fuzzy connected intensity and directed fuzzy connected intensity matrix in DFSN. According to the directed fuzzy connected intensity matrix, we can get the fuzzy relation between any of the two actors of the DFSN.

Then, we propose some new centrality measures: fuzzy in-degree centrality, fuzzy out-degree centrality and fuzzy degree centrality in this paper, which are applicable to DFSNs. The fuzzy out-degree centrality are measures of expansiveness and the fuzzy in-degree centrality are measures of receptivity, or popularity. If we consider the sociometric relation of friendship, an actor with a large fuzzy out-degree centrality is one who have many others as friends. An actor with a small fuzzy out-degree centrality nominates fewer friends. An actor with a large fuzzy in-degree centrality is one whom many others nominate as a friend, and an actor with a small fuzzy in-degree centrality is chosen by few others.

Besides, in this paper, we propose some new centrality measures: fuzzy in-closeness centrality, fuzzy out-closeness centrality and fuzzy closeness centrality which are applicable to the DFSNs. The major difference between fuzzy closeness centrality and fuzzy degree centrality lies in the fact that indirect fuzzy relationship is taken into consideration.

Fourthly, the mean fuzzy in-degree centrality and mean fuzzy out-degree centrality are discussed in this paper. In DFSN, the mean fuzzy in-degree centrality is equal to the mean fuzzy out-degree centrality.

In addition, we propose the variance of the fuzzy in-degree centrality, fuzzy out-degree centrality, fuzzy in-closeness centrality and fuzzy out-closeness centrality in this paper, which are applicable to the DFSNs. These measures quantify how unequal the actors in a DFSN are with respect to initiating or receiving fuzzy relations. These measures are simple statistics for summarizing how "centralized" a DFSN is.

And finally, we apply these measures to G-Y Research Team. We find that based on these measures, v_6 and v_5 get the two highest scores; because it has many direct and indirect fuzzy relations with the other researchers.

Fuzzy centrality analysis is one of the most important and commonly used tools in DFSN. This is a measurement concept concerning an actor's central position in the DFSN, and it reflects the different positions and advantages between DFSN actors. This study gives further supplement to the centrality theory and provide the theoretical foundation for further study of the DFSN.

Acknowledgments

We wish to give many thanks to Editor and anonymous referees for their valuable feedback on earlier versions of this paper. Thanks to the support by National Natural Science Foundation of China (No. 71173051, No.71403060), Ministry of Education Chang Jiang scholars and innovation team development plan (IRT1224), Guangdong Province Natural Science Foundation Research Team Project (S2013030015737), Humanities and Social Science Research Youth Fund Projects of Ministry of Education (13YJC630101), Philosophy Social Science Project of Guangdong Province (No. GD13YGL04), and Philosophy Social Science Project of Guangzhou (No.14Q22).

References

[1] S. Wasserman, K. Faust, Social Network Analysis: Methods and Applications, Cambridge University Press, Cambridge, 1994.

[2] M.E. Shaw, Group structure and the behavior of individuals in small groups, Journal of Psychology 1 (1954) 139-149.

[3] J. Nieminen, On the centrality in a directed graph, Social Science Research 4 (1973) 371-378.

[4] M.A. Beauchamp, An improved index of centrality, Behavioral Science 2 (1965) 161-163.

[5] G. Sabidussi, The centrality index of a graph, Psychometrika 4 (1966) 581-603.

[6] A. Bavelas, A mathematical model for small group structures, Human Organization 3 (1948) 16-30.

[7] L.C. Freeman, A set of measures of centrality based on betweenness, Sociometry 1 (1977) 35-41.

[8] K. Stephenson, M. Zelen, Rethinking centrality: Methods and applications, Social Networks 1 (1989) 1-37.

[9] P. Bonacich, Factoring and weighting approaches to status scores and clique detection, Journal of Mathematical Sociology 1 (1972) 113-120.

[10] P. Bonacich, Power and centrality: A family of measures, American Journal of Sociology 5 (1987) 1170-1182.

[11] M. Brunelli, M. Fedrizzi, Fuzzy *m*-ary adjacency relations in social network analysis: Optimization and consensus evaluation. Information Fusion 17 (2014) 36-45.

[12] K. Sohn, D. Kim, Zonal centrality measures and the neighborhood effect, Transportation Research Part A Policy and Practice 9 (2010) 733-743.

[13] A.M. Kermarrec, E.L. Merrer, B. Sericola, G. Trdan, Second order centrality: Distributed assessment of nodes criticity in complex networks, Computer Communications 5 (2011) 619-628.

[14] B.Y. Shen, W. Chen, Network centrality, absorptive capacity and innovation performance of SME, Science and Technology Management Research 3 (2007) 100-102.

[15] L. Ping, L.Y. Zong, Research on microblog information dissemination based on SNA centrality analysis–a case study with Sina microblog, Intelligence, Information Sharing 6 (2010) 92-97.

[16] X.Q. Qi, E. Fuller, Q. Wu, Y.Z. Wu, C.Q. Zhang, Laplacian centrality: A new centrality measure for weighted networks, Information Sciences 1 (2012) 240-253.

[17] M. Pozo, C. Manuel, E. Gonzlez-Arangena, G. Owen, Centrality in directed social networks: A game theoretic approach, Social Networks 3 (2011) 191-200.

[18] A. Landherr, B. Friedl, J. Heidemann, A critical review of centrality measures in social networks, Business & Information Systems Engineering 2(6) (2010) 371-385.

[19] P. S. Nair, S. Sarasamma, Data mining through fuzzy social network analysis, Proceedings of the 26th Annual Meeting of the North American Fuzzy Information Processing Society, 2007, pp. 251-255.

[20] L.P. Liao, R.J. Hu, On the definition and property analysis of fuzzy social network based on fuzzy graph, Journal of Guangdong University of Technology: Social Science Edition 3 (2012) 46-51.

[21] T.F. Fan, C.J. Liau, T.Y. Lin, Positional analysis in fuzzy social networks, Proceedings of the 3rd IEEE International Conference on Granular Computing, 2007, pp. 423-428.

[22] T.F. Fan, C.J. Liau, T.S. Lin, A theoretical investigation of regular equivalences for fuzzy graphs, International Journal of Approximate Reasoning 3 (2008) 678-688.

[23] L.P. Liao, R.J. Hu, G.Y. Zhang, The position analysis of the fuzzy technology innovation network, Journal of High Technology Management Research 2 (2012) 83-89.

[24] M.L. Tseng, Implementation and performance evaluation using the fuzzy network balanced scorecard, Computers Education 1 (2010) 188-201.

[25] L.P. Liao, R.J. Hu, G.Y. Zhang, The centrality analysis of fuzzy social network, Fuzzy Systems and Mathematics 2 (2012) 169-173.

[26] R.J. Hu, G.Y. Zhang, L.P. Liao, The closeness centrality analysis of fuzzy social network based on inversely attenuation factor, Fuzzy Information & Engineering and Operations Research & Management 211 (2014) 457-465.

[27] R.J. Hu, G.Y. Zhang, Structural holes in directed fuzzy social networks, Journal of Applied Mathematics 2014 (2014) 1-8.

[28] L.A. Zadeh, Fuzzy sets, Information and Control 3 (1965) 338-358.

[29] L.A. Zadeh, A fuzzy-algorithmic approach to the definition of complex or imprecise concepts, International Journal of Man-Machine Studies 8 (1976) 249-291.